Subject Code: RCS301

PAPER ID-311262

Roll No:

B TECH
(SEM-III) THEORY EXAMINATION 2020-21
DISCRETE STRUCTURES & THEORY OF LOGIC

Time: 3 Hours ***Total Marks: 70***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 7 = 14

a.	Prove that $(A \cup B)^c = A^c \cap B^c$
b.	How many antisymmetric relations can be from the set A containing n distinct elements?
c.	Prove that every cyclic group is an abelian group.
d.	Consider the group (Z, +). Let $H = \{3n: n \in Z\}$. Show that H is a subgroup of Z.
e.	Prove that every distributive lattice is modular.
f.	Find the generating function for the sequence $1, a, a^2, \ldots\ldots$ where a is a constant.
g.	Write the converse and contrapositive of the following statement: "I get success whenever I work hard."

SECTION B

2. Attempt any *three* of the following: 7 x 3 = 21

a.	If $f: A \to B$ and $g: B \to C$ be one-to-one and onto functions, then prove that gof is also one-to-one and onto function. Also prove that $(gof)^{-1} = f^{-1} o\, g^{-1}$
b.	Define subgroup. Prove that the necessary and sufficient condition for a non-empty subset H of a group (G, *) to be a subgroup is $a \in H, b \in H$ implies $a*b^{-1} \in H$.
c.	Prove the De-Morgan's law in Boolean algebra i.e. i. $(a + b)' = a' \cdot b'$ ii. $(a.b)' = a' + b'$
d.	Prove the validity of the following argument "if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed."
e.	Prove the following: i. In any binary tree T on n vertices, the number of pendant vertices is equal to $(n+1)/2$. ii. The number of internal vertices in a binary tree is one less than the number of pendant vertices.

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

(a)	Prove by mathematical induction for all positive integers that: $4^{2n+1} + 3^{n+2}$ is an integer multiple of 13.
(b)	Let A = {2, 3, 4, 5}. Then relation R and S on A defined by: R = {(2, 2), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (5, 3)} S = {(2, 3), (2, 5), (3, 4), (3, 5), (4, 2), (4, 3) (4, 5), (5, 2), (5, 5)} Find the matrices of the above relations. Use the matrices to find the following composition of the relation R and S.

4. Attempt any *one* part of the following: 7 x 1 = 7

(a)	Let $G = \{(a, b) \mid a, b \in R, a \neq 0\}$. Define a binary operation * on G by $(a, b) * (c, d) = (ac, bc + d)\ \forall\ (a, b), (c, d) \in G$. Show that (G, *) is a group.
(b)	State and prove Lagrange theorem for group.

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5. Attempt any *one* part of the following: **7 x 1 = 7**

(a)	Express the following in the disjunctive normal form: i. $F(x, y, z) = x(y'z)'$ ii. $F(x, y, z) = (x + y).(x + z) + y + z'$
(b)	Prove that the product of two lattices is a lattice.

6. Attempt any *one* part of the following: **7 x 1 = 7**

(a)	Show that s is valid conclusion, from the premises $p \Rightarrow q$, $p \Rightarrow r$, $\sim (q \wedge r)$ and $s \vee p$.
(b)	Show that $(p \to r) \wedge (q \to r)$ *and* $(p \vee q) \to r$ are logically equivalent.

7. Attempt any *one* part of the following: **7 x 1 = 7**

(a)	Solve the following recurrence relation: $a_{n+2} - 2a_{n+1} + a_n = 0$ where $a_0 = 2$, $a_1 = 1$
(b)	Explain the following: i. Regular graph ii. Complete graph iii. Multigraph iv. Bipartite graph v. Euler and Hamiltonian paths